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LXVII. Fig. 30.

$ISPK$ is equivalent to $2IFK=2ADB$ is equivalent to $2ACB+ACDE$ is equivalent to $ACB+FHQ+ACDE$.

$SABP$ is equivalent to $FABQ$.

$\therefore$ $ABKI$ is equivalent to $ACDE+BCFH$.

LXVIII. Fig. 30.

$ILR=ACD$, and $ILF=AED$.

Then $IRK=IFA$. $BHK=BCA$.

$\therefore$ $ABKI$ is equivalent to $ACDE+BCFH$.

LXIX. Fig. 30.

$LNOC$ is equivalent to $2LAC=2FED$ is equivalent to $ACDE$.

$KLCB$ is equivalent to $BCFH$.

$\therefore$ $ABKI$ is equivalent to $KNOB$ is equivalent to $ACDE+BCFH$.

[To be Continued.]

NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M. (Princeton); Ph. D. (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

[Continued from October Number.]

PROPOSITION XXIX. *Resuming Fig. 33 of the preceding proposition: I say every straight AC, which cuts angle BAX, finally at a finite or terminated distance (even in hypothesis of acute angle) will meet BX in a certain point P, if only AC be produced ever more toward the parts of the points X.*

Proof. And firstly indeed (lest straight AC include space with AX) it must meet at finite distance the straights LK, HK, DK in certain points C, N, M; must meet, I say, unless before (and that at a finite distance, just as we maintain) it meets BX in some point between the point B and one of the points K.

Then (from Corollary I. after XXIII.) the angles ACK, ANK, AMK will be obtuse.

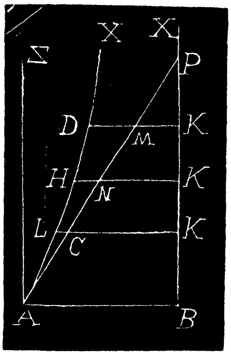

Fig. 33.

Moreover those angles, always obtuse, approach (from the preceding proposition) without any certain limit, to equality with a right angle, when indeed that AC is supposed to meet BX only at an infinite distance. Therefore such an ordinate KMD can be reached that at it the angle AMK

exceeds a right angle by less than the angle *DAC*. But then angle *DAC*, or *DAM*, together with angle *AMD* will be greater than a right angle. Wherefore, the obtuse angle *ADM* being added, the three angles together of the triangle *ADM* will be greater than two right angles, which is against the hypothesis of acute angle.

Therefore every straight *AC*, which cuts that angle *BAX*, finally at a finite or terminated distance (hypothesis of acute angle) must meet *BX* in a certain point *P*. Quod etc.

Corollary I. Hence no straight *AZ*, which toward the parts of the points *X* makes an acute angle greater than *BAX* can ever meet *BX*, either at a finite, or at an infinite distance. For as far as so should happen, now *AX* dividing angle *BAZ*, ought (against the premised supposition) to meet *BX* at a finite distance, as this is demonstrated of the straight *AC* dividing angle *BAX*.

Corollary II. Moreover it follows that no determinate acute angle will be the maximum of all under which a straight line produced from point *A* meets *BX* at finite distance. For if toward the parts of the point *X* you assume any point higher than the point *P*, it follows that the straight joining point *A* with this higher point will make with *AB* a greater angle than angle *BAP*. And so ever without any intrinsic end. Wherefore angle *BAX* (since indeed *AX* both always approaches to *BX*, and meets it only at an infinite distance) will be the outside limit of all acute angles under which straights produced from that point *A* meet the aforesaid *BX* at a finite distance.

[To be Continued.]

SOPHUS LIE'S TRANSFORMATION GROUPS.

A SERIES OF ELEMENTARY, EXPOSITORY ARTICLES.

By **EDGAR ODELL LOVETT**, Princeton University.

II.

The Group of One Parameter. The Infinitesimal Transformation. Existence of an Infinitesimal Transformation in a Group of One Parameter.

5. Consider the plane as a point manifoldness, *i. e*, a space whose space element is the point. The plane will then be two-dimensional,* *i. e* contain ∞^2

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*This idea and its bearing in the paragraph are emphasized here not to introduce any unnecessary ultra refinement but because of their use in geometrical illustrations to appear in succeeding articles. For example, the plane is one, two, three, or four dimensional according as, a circle with fixed center, the straight line, a circle of general position, or parabola, be taken as space element, since there are ∞^1, ∞^2, ∞^3, ∞^4, of these elements respectively in the plane. Similarly if the straight line is element it has no dimension, the point has one dimension, the plane two dimensions, and ordinary space four dimensions.